Stability in a Commonsense Ontology of States

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Abstract

Despite their mostly-unchallenged acceptance in a reasoning agent's ontology, states, as opposed to events, have raised some interesting questions over the years, particularly regarding the representation of their temporal incidence and the striking similarity they have to propositions. I present a family of languages, $Log_A\mathbf{S}$, for reasoning about states. A $Log_A\mathbf{S}$ language is algebraic in the sense that it does not contain sentences, only terms, some of which denoting states. In this paper, I study one particular aspect of the state ontology, namely temporal stability, classifying states according to their pattern of persistence over time. I identify four major intuitive stability classes which partition the domain of states and prove closure properties for each class under the various Boolean operators. One of these classes, the class of *atemporal states*, corresponds one-to-one to the set of propositions, thus providing a unified treatment of propositions and states.

Introduction

In discussions of aspectual phenomena, "states" stand as one prominent kind of entity of which temporal incidence is attributable (Vendler 1957; Herweg 1991; Smith 1999). As opposed to "events", states do not wholly "occur", but homogeneously "hold" over time. States appear in different guises and under different names in the artificial intelligence and related literature. They are the "states of affairs" of metaphysics (Textor 2012), the "partial states" of (Bennett and Galton 2004), the "properties" of (Allen 1984), and the fluents of the situation calculus (McCarthy and Hayes 1969) and some variants of the event calculus (Shanahan 1999). I have argued elsewhere (Ismail 2008) for the primitiveness of states in the commonsense ontology of an acting agent, being the sole objects of experience from which all of an agent's beliefs are constructed through inference. As such, a thorough investigation of states is at least insightful.

But states are not unanimously accepted as inhabitants of the temporal ontology. As indicated, they do appear as fluents in the situation calculus, and as properties in Allen's ontology (Allen 1984), for example. But these are all first-order theories. In tense logics (Prior 1967), for instance, we do not see any notion of states, only propositions which may be true in the present, the past, or the future. It is not clear whether first-order theories which endorse states do that out of conviction or as a side-effect of their attempt to secure the convenience of having state-denoting terms in the logical language, which facilitates tersely axiomatizing temporal properties. In particular, in first-order theories with state-denoting terms, there is always a sense of redundancy when the entire syntax for sentence formation is duplicated to form the set of state terms. (See (Shoham 1987) who criticizes (Allen 1984) particularly for this point.) The mysterious syntactic division between state terms and sentences, and the corresponding semantic division between states and propositions, is unsettling since it is (i) theoretically suspicious, and (ii) both syntactically and ontologically uneconomical.

In this paper, I present $Log_A\mathbf{S}$, a family of algebraic logics of states. $Log_A\mathbf{S}$ is algebraic in the sense that it only contains terms, algebraically constructed from function symbols. No sentences are included in a $Log_A\mathbf{S}$ language. Instead, there are terms of a distinguished syntactic type that are taken to denote states; some of these terms may, for convenience, be regarded as sentences. $Log_A\mathbf{S}$ is a variant of $Log_A\mathbf{B}$ (Ismail 2012), which is, again, an algebraic language for reasoning about propositions and propositional attitudes. The main difference between $Log_A\mathbf{B}$ and $Log_A\mathbf{S}$ is that whereas the ontology of the former includes propositions, taken at face value, the ontology of the latter trades propositions for states.

In the $Log_A\mathbf{S}$ ontology, states are structured in a Boolean algebra. This gives us, almost for free, state-counterparts to propositional truth conditions and standard notions of consequence and validity. Here, I investigate one particular aspect of the state ontology, namely temporal stability, classifying states according to their pattern of persistence over time. Four major classes are identified, disjointness between four of the classes, and closure properties of each class under the Boolean operators are proved. As it turns out, the class of *atemporal states* corresponds one-to-one to the set of propositions. Thus, through its state-based syntax and semantics, $Log_A\mathbf{S}$ provides a unified, non-redundant treatment of states and propositions.

$Log_A\mathbf{S}$ Languages

$Log_A\mathbf{S}$ is a class of many-sorted languages that share a common core of logical symbols and differ in a signature of non-logical symbols. In what follows, we identify a sort σ with the set of symbols of sort σ. A $Log_A\mathbf{S}$ language is a set of terms partitioned into three base syntactic types, σ_S, σ_T, and σ_I. Intuitively, σ_S is the set of terms denoting states, σ_T is the set of terms denoting time points, and σ_I is the set of terms denoting anything else.

Syntax

As is customary in type-theoretical treatments, an alphabet of $Log_A\mathbf{S}$ is made up of a set of syncategorematic punctuation symbols and a set of denoting symbols each from a set σ of syntactic types. The set σ is the smallest set containing all of the following types: σ_S, σ_T, σ_I, and $\varsigma_1 \longrightarrow \varsigma_2$, for $\varsigma_1 \in \{\sigma_S, \sigma_T, \sigma_I\}$ and $\varsigma_2 \in \sigma$. Intuitively, $\varsigma_1 \longrightarrow \varsigma_2$ is the syntactic type of function symbols that take a single argument of type σ_S, σ_T, or σ_I and produce a functional term of type ς_2. Given the restriction of the first argument of function symbols to base types, $Log_A\mathbf{S}$ is, in a sense, a first-order language.

A $Log_A\mathbf{S}$ alphabet is a union of four disjoint sets: $\Omega \cup \Xi \cup \Sigma \cup \Lambda$. The set Ω, the *signature* of the language, is a non-empty set of constant and function symbols. Each symbol in the signature has a designated syntactic type from σ. Ω is what distinguishes one $Log_A\mathbf{S}$ language from another. The set $\Xi = \{x_i, t_i, s_i\}_{i\in\mathbb{N}}$ is a countably infinite set of variables, where $x_i \in \sigma_I$, $t_i \in \sigma_T$, and $s_i \in \sigma_S$. Σ is a set of syncategorematic symbols, including the comma, various matching pairs of brackets and parentheses, and the symbol $\forall$. The set Λ is the set of logical symbols of $Log_A\mathbf{S}$, defined as the union of the following sets.

1. $\neg \in \sigma_S \longrightarrow \sigma_S$
2. $\{\wedge, \vee\} \subseteq \sigma_S \longrightarrow \sigma_S \longrightarrow \sigma_S$
3. $\mathbf{HoldsAt} \in \sigma_S \longrightarrow \sigma_T \longrightarrow \sigma_S$
4. $\prec \in \sigma_T \longrightarrow \sigma_T \longrightarrow \sigma_S$

A $Log_A\mathbf{S}$ language with signature Ω is denoted by L_Ω. It is the smallest set of terms formed according to the following rules, where τ and τ_i ($i \in \mathbb{N}$) are terms in L_Ω.

- $\Xi \subset L_\Omega$
- $c \in L_\Omega$, where $c \in \Omega$ is a constant symbol.
- $f(\tau_1, \ldots, \tau_n) \in L_\Omega$, where $f \in \Omega$ is of type $\varsigma_1 \longrightarrow \ldots \longrightarrow \varsigma_n \longrightarrow \varsigma$ ($n > 0$) and τ_i is of type ς_i.
- $\neg\tau \in L_\Omega$, where $\tau \in \sigma_S$.
- $(\tau_1 \otimes \tau_2) \in L_\Omega$, where $\otimes \in \{\wedge, \vee\}$ and $\tau_1, \tau_2 \in \sigma_S$.
- $\forall x(\tau) \in L_\Omega$, where $x \in \Xi$ and $\tau \in \sigma_S$.
- $\mathbf{HoldsAT}(\tau_1, \tau_2) \in L_\Omega$, where $\tau_1 \in \sigma_S$ and $\tau_2 \in \sigma_T$.
- $(\tau_1 \prec \tau_2) \in L_\Omega$, where $\tau_1, \tau_1 \in \sigma_T$.

As usual, terms involving $\Rightarrow$, $\Leftrightarrow$, and $\exists$ may be introduced as abbreviations in the standard way.

Semantics

The basic ingredient of the $Log_A\mathbf{S}$ semantic apparatus is the notion of a $Log_A\mathbf{S}$ structure.

Definition 1 *A* $Log_A\mathbf{S}$ structure *is a quadruple* $\mathfrak{S} = \langle \mathcal{D}, \mathfrak{A}, \mathfrak{h}, < \rangle$, *where*

- $\mathcal{D}$*, the domain of discourse, is a set with two disjoint, non-empty, countable subsets* $\mathcal{S}$ *and* $\mathcal{T}$.
- $\mathfrak{A} = \langle \mathcal{S}, +, \cdot, -, \bot, \top \rangle$ *is a complete (closed under arbitrary products and sums), non-degenerate* ($\top \neq \bot$) *Boolean algebra.*
- $\mathfrak{h} : \mathcal{S} \times \mathcal{T} \longrightarrow \mathcal{S}$ *satisfies the following properties, for every* $\hat{S} \subseteq S$, $s \in \mathcal{S}$ *and* $t, t_1, t_2 \in \mathcal{T}$:
  1. $\mathfrak{h}(-s, t) = -\mathfrak{h}(s, t)$.
  2. $\mathfrak{h}(\prod_{s\in\hat{S}} s, t) = \prod_{s\in\hat{S}} \mathfrak{h}(s, t)$.
  3. $\mathfrak{h}(\mathfrak{h}(s, t_1), t_2) = \mathfrak{h}(s, t_1)$.
  4. *If* $\prod_{t\in\mathcal{T}} \mathfrak{h}(s, t) = \top$, *then* $s = \top$.
- $<: \mathcal{T} \times \mathcal{T} \longrightarrow \mathcal{S}$ *defines an irreflexive linear order on* $\mathcal{T}$ *which is serial in both directions; that is,* $<$ *is constrained as follows, for every distinct* $t_1, t_2, t_3 \in \mathcal{T}$:
  1. $t_1 < t_2 = -(t_2 < t_1)$.
  2. $[(t_1 < t_2) \cdot (t_2 < t_3)] + t_1 < t_3 = t_1 < t_3$.
  3. $t_1 < t_1 = \bot$.
  4. $\sum_{t\in\mathcal{T}} t_1 < t = \sum_{t\in\mathcal{T}} t < t_1 = \top$.
- *For* $t_1, t_2, t_3 \in \mathcal{T}$, $\mathfrak{h}(t_1 < t_2, t_3) = t_1 < t_2$.

Intuitively, the domain $\mathcal{D}$ is partitioned by a set of states $\mathcal{S}$, structured as a Boolean algebra, a set of linearly-ordered time points $\mathcal{T}$, and a set of individuals $\overline{\mathcal{S} \cup \mathcal{T}}$. The function $\mathfrak{h}$ maps a state s and a time point t to the state of s's holding at t. The first two properties of $\mathfrak{h}$ indicate that it commutes with the Boolean operators. (cf. (Allen 1984).) The third property secures the intuition that the state $\mathfrak{h}(s, t)$ of s's holding at t preserves its identity under further applications of $\mathfrak{h}$. (More on this below.) The final property of $\mathfrak{h}$ requires that the only state which necessarily holds at all times is $\top$. The following observations complete the characterization of $\mathfrak{h}$.

Observation 1 $\mathfrak{h}(\sum_{s\in\hat{S}}, t) = \sum_{s\in\hat{S}} \mathfrak{h}(s_2, t)$.

Observation 2 *If* $\sum_{t\in\mathcal{T}} \mathfrak{h}(s, t) = \bot$, *then* $s = \bot$.

The three parts of $\mathcal{D}$ stand in correspondence to the syntactic sorts of $Log_A\mathbf{S}$. In what follows, we let $\mathcal{D}_{\sigma_S} = \mathcal{S}$, $\mathcal{D}_{\sigma_T} = \mathcal{T}$, and $\mathcal{D}_{\sigma_I} = \overline{\mathcal{S} \cup \mathcal{T}}$.

Definition 2 *A* valuation $\mathcal{V}$ *of a* $Log_A\mathbf{S}$ *language* L_Ω *is a triple* $\langle \mathfrak{S}, \mathcal{V}_\Omega, \mathcal{V}_\Xi \rangle$, *where*

- $\mathfrak{S} = \langle \mathcal{D}, \mathfrak{A}, \mathfrak{h}, < \rangle$ *is a* $Log_A\mathbf{S}$ *structure;*
- $\mathcal{V}_\Omega$ *is a function that assigns to each constant of sort* ς *in* Ω *an element of* $\mathcal{D}_\varsigma$, *and to each function symbol* $f \in \Omega$ *of sort* $\varsigma_1 \longrightarrow \ldots \longrightarrow \varsigma_n \longrightarrow \varsigma$ *an* n*-adic function* $\mathcal{V}_\Omega(f) : \bigtimes_{i=1}^{n} \mathcal{D}_{\varsigma_i} \longrightarrow \mathcal{D}_\varsigma$*; and*
- $\mathcal{V}_\Xi : \Xi \longrightarrow \mathcal{D}$ *is a function (a* variable assignment*), where for every* $x \in \Xi$, *if* $x \in \varsigma$ *then* $v_\Xi(x) \in \mathcal{D}_\varsigma$.

In what follows, for a valuation $\mathcal{V} = \langle \mathfrak{S}, \mathcal{V}_\Omega, \mathcal{V}_\Xi \rangle$ with $x \in \Xi$ of sort ς and $a \in \mathcal{D}_\varsigma$, $\mathcal{V}[a/x] = \langle \mathfrak{S}, \mathcal{V}_\Omega, \mathcal{V}_\Xi[a/x] \rangle$, where $\mathcal{V}_\Xi[a/x](x) = a$, and $\mathcal{V}_\Xi[a/x](y) = \mathcal{V}_\Xi(y)$ for every $y \neq x$.

Definition 3 *Let L_Ω be a $Log_A\mathbf{S}$ language and let $\mathcal{V}$ be a valuation of L_Ω. An* interpretation *of the terms of L_Ω is given by a function $[\![\cdot]\!]^{\mathcal{V}}$:*

- $[\![x]\!]^{\mathcal{V}} = \mathcal{V}_\Xi(x)$, *for* $x \in \Xi$
- $[\![c]\!]^{\mathcal{V}} = \mathcal{V}_\Omega(c)$, *for a constant* $c \in \Omega$
- $[\![f(\tau_1, \ldots, \tau_n)]\!]^{\mathcal{V}} = \mathcal{V}_\Omega(f)([\![\tau_1]\!]^{\mathcal{V}}, \ldots, [\![\tau_n]\!]^{\mathcal{V}})$, *for an n-adic ($n \geq 1$) function symbol $f \in \Omega$*
- $[\![(\tau_1 \wedge \tau_2)]\!]^{\mathcal{V}} = [\![\tau_1]\!]^{\mathcal{V}} \cdot [\![\tau_2]\!]^{\mathcal{V}}$
- $[\![(\tau_1 \vee \tau_2)]\!]^{\mathcal{V}} = [\![\tau_1]\!]^{\mathcal{V}} + [\![\tau_2]\!]^{\mathcal{V}}$
- $[\![\neg\tau]\!]^{\mathcal{V}} = -[\![\tau]\!]^{\mathcal{V}}$
- $[\![\forall x(\tau)]\!]^{\mathcal{V}} = \prod_{a \in \mathcal{D}_\varsigma} [\![\tau]\!]^{\mathcal{V}[a/x]}$, *where x is of sort ς*
- $[\![\mathbf{HoldsAT}(\tau_1, \tau_2)]\!]^{\mathcal{V}} = \mathfrak{h}([\![\tau_1]\!]^{\mathcal{V}}, [\![\tau_2]\!]^{\mathcal{V}})$
- $[\![(\tau_1 \prec \tau_2)]\!]^{\mathcal{V}} = [\![\tau_1]\!]^{\mathcal{V}} < [\![\tau_2]\!]^{\mathcal{V}}$

In $Log_A\mathbf{S}$, logical consequence is defined in pure algebraic terms without alluding to the notion of truth, or to the holding of states. This is achieved using the natural partial order $\leq$ associated with $\mathfrak{A}$ (Burris and Sankappanavar 1982), where, for $s_1, s_2 \in \mathcal{S}$, $s_1 \leq s_2 =_{\text{def}} s_1 \cdot s_2 = s_1$. (Alternatively, $s_1 \leq s_2 =_{\text{def}} s_1 + s_2 = s_2$.)

Definition 4 *Let L_Ω be a $Log_A\mathbf{S}$ language. For every $\phi \in \sigma_S$ and $\Gamma \subseteq \sigma_S$, ϕ is a logical consequence of Γ, denoted $\Gamma \models \phi$, if, for every L_Ω valuation $\mathcal{V}$, $\prod_{\gamma \in \Gamma} [\![\gamma]\!]^{\mathcal{V}} \leq [\![\phi]\!]^{\mathcal{V}}$.*

Intuitively, ϕ is a logical consequence of Γ if ϕ is already *part* of Γ, so that their conjunction is nothing more than Γ itself. This *implies* that, should Γ holds, ϕ necessarily does; but it is not *defined* as such. By the above definition, and the algebraic properties of $\mathfrak{S}$, we can easily verify the validity of the following typical examples of logical consequence:

$$\{\phi \wedge \psi\} \models \phi, \{\phi\} \models \phi \vee \psi, \{\phi \Rightarrow \psi, \phi\} \models \psi, \{\bot\} \models \phi$$

In (Ismail 2012), it is shown that $\models$ has the distinctive properties of classical Tarskian logical consequence and that it satisfies a counterpart of the deduction theorem.

Stability

One dimension along which to classify states, is that of *temporal stability.* Temporal stability refers to the tendency, or lack thereof, of a state to change from holding to not holding, and the general patterns of such changes. The most stable states are what I call *eternal states*.

Eternal states do not start or cease; they either *always* hold or *never* hold (cf. Quine's *eternal sentences* (Quine 1960).) Examples of eternal states include whales' being fish (yes, *fish*), God's existence, and the state of the date of John's graduation being June 1st, 2001.

Definition 5 *$s \in \mathcal{S}$ is eternal if $\sum_{t \in \mathcal{T}} \mathfrak{h}(s,t) \leq \prod_{t \in \mathcal{T}} \mathfrak{h}(s,t)$.*

It follows from the above definition that, if s is eternal, $\mathfrak{h}(s, t_1) = \mathfrak{h}(s, t_2)$, for any $t_1, t_2 \in \mathcal{T}$. Thus, the notion of an eternal state's holding at a particular time is rendered meaningless, or redundant; an eternal state simply holds (or not), period. Hence, it is not clear what exactly distinguishes an eternal state s from a state $\mathfrak{h}(s,t)$. In the final analysis, it might turn out that, in general, such states are distinct; however, we are lead to identify the following important subset.

Definition 6 *$s \in \mathcal{S}$ is atemporal if $\mathfrak{h}(s,t) = s$, for every $t \in \mathcal{T}$.*

In what follows, let ETER and ATEMP denote the set of eternal states and the set of atemporal states, respectively.

Corollary 1 *$Range(\mathfrak{h}) \cup Range(<) \subseteq$ ATEMP $\subseteq$ ETER.*

Theorem 1

1. $\bot \in$ ETER(ATEMP).

2. $\top \in$ ETER(ATEMP).

3. If $s \in$ ETER(ATEMP), then $-s \in$ ETER(ATEMP).

4. If $\hat{S} \subseteq$ ETER(ATEMP), then $\{\sum_{s \in \hat{S}} s, \prod_{s \in \hat{S}} s\} \subseteq$ ETER(ATEMP).

Proof. We present the proof for the case of ETER.

1. Follows from Definition 1 and Corollary 1.
2. Follows from Observation 1 and Corollary 1.
3. Let $s \in$ ETER. Now,

$$\sum_{t \in \mathcal{T}} \mathfrak{h}(-s,t) = \sum_{t \in \mathcal{T}} -\mathfrak{h}(s,t) = -\prod_{t \in \mathcal{T}} \mathfrak{h}(s,t)$$
$$\leq -\sum_{t \in \mathcal{T}} \mathfrak{h}(s,t) = \prod_{t \in \mathcal{T}} -\mathfrak{h}(s,t) = \prod_{t \in \mathcal{T}} \mathfrak{h}(-s,t)$$

Hence, $-s \in$ ETER.

4. Let $\hat{S} \subseteq$ ETER. Hence,

$$\sum_{t \in \mathcal{T}} \mathfrak{h}(\sum_{s \in \hat{S}} s, t) = \sum_{t \in \mathcal{T}} \sum_{s \in \hat{S}} \mathfrak{h}(s,t) = \sum_{s \in \hat{S}} \sum_{t \in \mathcal{T}} \mathfrak{h}(s,t)$$
$$\leq \sum_{s \in \hat{S}} \prod_{t \in \mathcal{T}} \mathfrak{h}(s,t) \leq \sum_{s \in \hat{S}} \prod_{t \in \mathcal{T}} \mathfrak{h}(\sum_{\hat{s} \in \hat{S}} \hat{s}, t)$$
$$= \prod_{t \in \mathcal{T}} \mathfrak{h}(\sum_{\hat{s} \in \hat{S}} \hat{s}, t) = \prod_{t \in \mathcal{T}} \mathfrak{h}(\sum_{s \in \hat{S}} s, t)$$

Thus, $\sum_{s \in \hat{S}} s \in$ ETER. Moreover, $\prod_{s \in \hat{S}} s \in$ ETER:

$$\sum_{t \in \mathcal{T}} \mathfrak{h}(\prod_{s \in \hat{S}} s, t) = \sum_{t \in \mathcal{T}} \prod_{s \in \hat{S}} \mathfrak{h}(s,t) \leq \prod_{s \in \hat{S}} \sum_{t \in \mathcal{T}} \mathfrak{h}(s,t)$$
$$\leq \prod_{s \in \hat{S}} \prod_{t \in \mathcal{T}} \mathfrak{h}(s,t) = \prod_{t \in \mathcal{T}} \prod_{s \in \hat{S}} \mathfrak{h}(s,t) = \prod_{t \in \mathcal{T}} \mathfrak{h}(\prod_{s \in \hat{S}} s, t)$$

□

Thus, both ETER and ATEMP are closed under the Boolean operations.

Corollary 2 *$\langle$ETER(ATEMP)$, +, \cdot, -, \bot, \top\rangle$ is a subalgebra of $\langle \mathcal{S}, +, \cdot, -, \bot, \top \rangle$.*

The class of *permanent* states shares some of the stability of ETER. Unlike eternal states, permanent states may *start* to hold; once a permanent state starts to hold, however, it never ceases. The prototypical example of a permanent state is the perfect state of an event having occurred (Galton 1984). Other examples may include the state of Fermat's being dead or the state of my holding a Ph.D.

Definition 7 *A state $s \in \mathcal{S}$ is permanent if*

1. $s \neq \bot$,
2. $\prod_{t \in \mathcal{T}} \mathfrak{h}(s,t) = \bot$, *and*
3. *for every* $t_1, t_2 \in \mathcal{T}$, $[\mathfrak{h}(s,t_1) \cdot (t_1 < t_2)] \leq \mathfrak{h}(s,t_2)$.

In the sequel, PERM denotes the set of permanent states.

Theorem 2 PERM $\cap$ ETER $= \varnothing$.

Proof. Suppose not. Then, there is some $s \in$ PERM $\cap$ ETER. Since $s \in$ ETER, then $\sum_{t \in \mathcal{T}} \mathfrak{h}(s,t) \leq \prod_{t \in \mathcal{T}} \mathfrak{h}(s,t)$. But since $s \in$ PERM, then $\sum_{t \in \mathcal{T}} \mathfrak{h}(s,t) \leq \prod_{t \in \mathcal{T}} \mathfrak{h}(s,t) = \bot$. Thus, $\sum_{t \in \mathcal{T}} \mathfrak{h}(s,t) = \bot$. By Observation 2, it follows that $s = \bot$, which is impossible since $\bot \notin$ PERM. □

Observation 3 *For every* $s \in$ PERM *and* $t_1, t_2 \in \mathcal{T}$, $[\mathfrak{h}(-s,t_1) \cdot (t_2 < t_1)] \leq \mathfrak{h}(-s,t_2)$.

Theorem 3 *If* $s \in$ PERM, *then* $-s \notin$ PERM.

Proof. Assume that $-s, s \in$ PERM. Thus,

$$\begin{aligned}
\top = \top \cdot \top &= \sum_{t_1 \in \mathcal{T}} \mathfrak{h}(s,t_1) \cdot \sum_{t_2 \in \mathcal{T}} \mathfrak{h}(-s,t_2) \\
&= \sum_{t_1 \in \mathcal{T}} \sum_{t_2 \in \mathcal{T}} \mathfrak{h}(s,t_1) \cdot \mathfrak{h}(-s,t_2) \\
&= [\sum_{t_2 \neq t_1 \in \mathcal{T}} \sum_{t_2 \in \mathcal{T}} \mathfrak{h}(s,t_1) \cdot \mathfrak{h}(-s,t_2)] \\
&\quad + [\sum_{t_3 \in \mathcal{T}} \mathfrak{h}(s,t_3) \cdot \mathfrak{h}(-s,t_3)] \\
&= [\sum_{t_2 \neq t_1 \in \mathcal{T}} \sum_{t_2 \in \mathcal{T}} \mathfrak{h}(s,t_1) \cdot \mathfrak{h}(-s,t_2)] + [\sum_{t_3 \in \mathcal{T}} \mathfrak{h}(\bot,t_3)] \\
&= [\sum_{t_2 \neq t_1 \in \mathcal{T}} \sum_{t_2 \in \mathcal{T}} \mathfrak{h}(s,t_1) \cdot \mathfrak{h}(-s,t_2)] + [\sum_{t_3 \in \mathcal{T}} \bot] \\
&= \sum_{t_2 \neq t_1 \in \mathcal{T}} \sum_{t_2 \in \mathcal{T}} \mathfrak{h}(s,t_1) \cdot \mathfrak{h}(-s,t_2) \\
&= \sum_{t_2 \neq t_1 \in \mathcal{T}} \sum_{t_2 \in \mathcal{T}} [\mathfrak{h}(s,t_1) \cdot \mathfrak{h}(-s,t_2)] \cdot \top \\
&= \sum_{t_2 \neq t_1 \in \mathcal{T}} \sum_{t_2 \in \mathcal{T}} [\mathfrak{h}(s,t_1) \cdot \mathfrak{h}(-s,t_2)] \\
&\qquad \cdot [(t_1 < t_2) + (t_2 < t_1)] \\
&= \sum_{t_2 \neq t_1 \in \mathcal{T}} \sum_{t_2 \in \mathcal{T}} [\mathfrak{h}(s,t_1) \cdot \mathfrak{h}(-s,t_2) \cdot (t_1 < t_2))] \\
&\qquad + [\mathfrak{h}(s,t_1) \cdot \mathfrak{h}(-s,t_2) \cdot (t_2 < t_1)] \\
&\leq \sum_{t_2 \neq t_1 \in \mathcal{T}} \sum_{t_2 \in \mathcal{T}} [\mathfrak{h}(s,t_2) \cdot \mathfrak{h}(-s,t_2)] \\
&\qquad + [\mathfrak{h}(s,t_1) \cdot \mathfrak{h}(-s,t_1)] \\
&= \sum_{t_2 \neq t_1 \in \mathcal{T}} \sum_{t_2 \in \mathcal{T}} \mathfrak{h}(\bot,t_2) + \mathfrak{h}(\bot,t_1) = \bot
\end{aligned}$$

But then $\top = \bot$, which is impossible since $\mathfrak{A}$ is non-degenerate. □

In what follows, let CO-PERM denote the set of complements of states in PERM:

$$\text{CO-PERM} = \{s \mid -s \in \text{PERM}\}$$

By duality, the prototypical example of a CO-PERM state is the state of an event's occurrence being in the future.

Observation 4 *A state $s \in$ CO-PERM if and only if*

1. $s \neq \top$,
2. $\sum_{t \in \mathcal{T}} \mathfrak{h}(s,t) = \top$, *and*
3. *for every* $t_1, t_2 \in \mathcal{T}$, $[\mathfrak{h}(s,t_1) \cdot (t_2 < t_1)] \leq \mathfrak{h}(s,t_2)$.

Theorem 4 *For every non-empty, finite* $\hat{S} \subseteq$ PERM, *if* $\prod_{s \in \hat{S}} s \neq \bot$ *then* $\{\sum_{s \in \hat{S}} s, \prod_{s \in \hat{S}} s\} \subseteq$ PERM.

Proof. We only prove that $\sum_{s \in \hat{S}} s \in$ PERM; the proof for products is simpler. We proceed by induction on the size n of $\hat{S}$.

Basis. For $\hat{S} = \{\hat{s}\}$, $\sum_{s \in \hat{S}} s = \hat{s}$, which is permanent.

Induction Hypothesis. For any $\hat{S} \subseteq$ PERM, with $|\hat{S}| \leq n$, $\sum_{s \in \hat{S}} s \in$ PERM.

Induction Step. Let $\hat{S} \subseteq$ PERM with $|\hat{S}| = n + 1$. Pick some $\hat{s_1} \in \hat{S}$, and let $\hat{s_2} = \sum_{s \in \hat{S} - \{\hat{s_1}\}} s$. By the induction hypothesis, $\hat{s_2} \in$ PERM. We need to show that $\sum_{s \in \hat{S}} s = \hat{s_1} + \hat{s_2} \in$ PERM.

1. Assume that $\hat{s_1} + \hat{s_2} = \bot$. Hence, $\hat{s_1} = \hat{s_2} = \bot$, which is impossible since $\bot \notin$ PERM.
2. In what follows, let $\phi = -(s_1 + s_2)$ and $\Phi = \sum_{t \in \mathcal{T}} \mathfrak{h}(\phi, t)$. We need to show that $\Phi = \top$. Hence,

$$\begin{aligned}
\top = \top \cdot \top &= \sum_{t_1 \in \mathcal{T}} \mathfrak{h}(-s_1,t_1) \cdot \sum_{t_2 \in \mathcal{T}} \mathfrak{h}(-s_2,t_2) \\
&= \sum_{t_1 \in \mathcal{T}} \sum_{t_2 \in \mathcal{T}} \mathfrak{h}(-s_1,t_1) \cdot \mathfrak{h}(-s_2,t_2) \\
&= [\sum_{t_2 \neq t_1 \in \mathcal{T}} \sum_{t_2 \in \mathcal{T}} \mathfrak{h}(-s_1,t_1) \cdot \mathfrak{h}(-s_2,t_2)] \\
&\quad + [\sum_{t_3 \in \mathcal{T}} \mathfrak{h}(-s_1,t_3) \cdot \mathfrak{h}(-s_2,t_3)] \\
&= [\sum_{t_2 \neq t_1 \in \mathcal{T}} \sum_{t_2 \in \mathcal{T}} \mathfrak{h}(-s_1,t_1) \cdot \mathfrak{h}(-s_2,t_2)] + \Phi \\
&= [\sum_{t_2 \neq t_1 \in \mathcal{T}} \sum_{t_2 \in \mathcal{T}} \mathfrak{h}(-s_1,t_1) \cdot \mathfrak{h}(-s_2,t_2) \\
&\qquad \cdot [(t_1 < t_2) + (t_2 < t_1)]] + \Phi \\
&= [\sum_{t_2 \neq t_1 \in \mathcal{T}} \sum_{t_2 \in \mathcal{T}} [\mathfrak{h}(-s_1,t_1) \cdot \mathfrak{h}(-s_2,t_2) \cdot (t_1 < t_2))] \\
&\quad + [\mathfrak{h}(-s_1,t_1) \cdot \mathfrak{h}(-s_2,t_2) \cdot (t_2 < t_1)]] + \Phi \\
&\leq [\sum_{t_2 \neq t_1 \in \mathcal{T}} \sum_{t_2 \in \mathcal{T}} [\mathfrak{h}(-s_1,t_1) \cdot \mathfrak{h}(-s_2,t_1)] \\
&\quad + [\mathfrak{h}(-s_1,t_2) \cdot \mathfrak{h}(-s_2,t_2)]] + \Phi \\
&= [\sum_{t_1 \in \mathcal{T}} \sum_{t_2 \in \mathcal{T}} \mathfrak{h}(\phi,t_1) + \mathfrak{h}(\phi,t_2)] + \Phi \\
&= \Phi + \Phi + \Phi = \Phi
\end{aligned}$$

   But since $\Phi \leq \top$, it follows that $\Phi = \top$.
3. Let $t_1, t_2 \in \mathcal{T}$. Thus,
   $\mathfrak{h}(s_1 + s_2, t_1) \cdot (t_1 < t_2)$

$$\begin{aligned}
&= [\mathfrak{h}(s_1,t_1) + \mathfrak{h}(s_2,t_1)] \cdot (t_1 < t_2) \\
&= [\mathfrak{h}(s_1,t_1) \cdot (t_1 < t_2)] + [\mathfrak{h}(s_2,t_1) \cdot (t_1 < t_2)] \\
&\leq \mathfrak{h}(s_1,t_2) + \mathfrak{h}(s_2,t_2) \\
&= \mathfrak{h}(s_1 + s_2, t_2)
\end{aligned}$$

□

Theorem 5 *For every non-empty, finite $\hat{S} \subseteq$ CO-PERM, if $\sum_{s\in\hat{S}} s \neq \top$ then $\{\sum_{s\in\hat{S}} s, \prod_{s\in\hat{S}} s\} \subseteq$ CO-PERM.*

Proof. Since $\sum_{s\in\hat{S}} s \neq \top$, then $\prod_{s\in\hat{S}} -s \neq \bot$. By Theorem 4, $\prod_{s\in\hat{S}} -s \in$ PERM. Hence, $\sum_{s\in\hat{S}} s = -\prod_{s\in\hat{S}} -s \in$ CO-PERM. The same follows, *mutatis mutandis*, for $\prod_{s\in\hat{S}}$. □

Hence, both PERM and CO-PERM are closed under non-trivial, finite products and sums, but not under complementation. Infinite sums and products are, in general, not (co-)permanence-preserving. For example, for any time t, the state that t is in the past is permanent. However, the state that *some* time is in the past, which is an infinite sum of PERM states, is eternal, given the left-seriality of $<$.

Definition 8 *A state s is temporary if $s \in$ TEMP $= \mathcal{S} -$ (ETER $\cup$ PERM $\cup$ CO-PERM).*

Theorem 6 *If $s \in$ TEMP then $-s \in$ TEMP.*

Temporary states are totally unconstrained and may, thus, contingently exhibit patterns of holding similar to those of ETER, PERM, and CO-PERM. However, some states strictly resist the anti-temporary patterns. For example, the state of an event being in progress is a temporary state which will never hold eternally, permanently, or co-permanently, since an event necessarily starts and ends (Ismail 2008).

Definition 9 *A state s is transient if*

1. $\sum_{t_1\in\mathcal{T}}\sum_{t_2\in\mathcal{T}}[(t_1 < t_2)\cdot\mathfrak{h}(s,t_1)\cdot -\mathfrak{h}(s,t_2)] = \top$, *and*
2. $\sum_{t_1\in\mathcal{T}}\sum_{t_2\in\mathcal{T}}[(t_2 < t_1)\cdot\mathfrak{h}(s,t_1)\cdot -\mathfrak{h}(s,t_2)] = \top$

In what follows, TRANS denotes the set of transient states.

Theorem 7 TRANS $\subseteq$ TEMP.

Proof. We prove the statement by demonstrating that TRANS $\cap$ [ETER $\cup$ PERM $\cup$ CO-PERM] $= \varnothing$.

1. Assume that $s \in$ TRANS $\cap$ ETER. Given Definition 5, $\sum_{t\in\mathcal{T}}\mathfrak{h}(s,t) \leq \prod_{t\in\mathcal{T}}\mathfrak{h}(s,t)$. But since $s \in$ TRANS then, by definition, $\top = \sum_{t_1\in\mathcal{T}}\sum_{t_2\in\mathcal{T}}[(t_1 < t_2)\cdot \mathfrak{h}(s,t_1)\cdot -\mathfrak{h}(s,t_2)]$. Thus,
$$\begin{aligned}\top &\leq [\sum_{t_1\in\mathcal{T}}\mathfrak{h}(s,t_1)]\cdot[\sum_{t_2\in\mathcal{T}}-\mathfrak{h}(s,t_2)]\\ &\leq [\prod_{t_1\in\mathcal{T}}\mathfrak{h}(s,t_1)]\cdot[\sum_{t_2\in\mathcal{T}}-\mathfrak{h}(s,t_2)]\\ &= \sum_{t_2\in\mathcal{T}}[-\mathfrak{h}(s,t_2)\cdot\prod_{t_1\in\mathcal{T}}\mathfrak{h}(s,t_1)] \quad = \sum_{t_2\in\mathcal{T}}\bot = \bot\end{aligned}$$
But this is impossible since $\mathfrak{A}$ is non-degenerate.
2. Assume that $s \in$ TRANS $\cap$ PERM. Hence, by Definition 7, for any $t_1, t_2 \in \mathcal{T}$, $[\mathfrak{h}(s,t_1)\cdot(t_1 < t_2)] \leq \mathfrak{h}(s,t_2)$. But, then, it follows that
$$\top \leq \sum_{t_1\in\mathcal{T}}\sum_{t_2\in\mathcal{T}}[\mathfrak{h}(s,t_2)\cdot -\mathfrak{h}(s,t_2)] = \sum_{t_1\in\mathcal{T}}\sum_{t_2\in\mathcal{T}}\bot = \bot.$$
Again, this is impossible since $\mathfrak{A}$ is non-degenerate.
3. Similar to the above case, TRANS $\cap$ CO-PERM $= \varnothing$.

□

Theorem 8 *If $s \in$ TRANS then $-s \in$ TRANS.*

Proof. Let $s \in$ TRANS. Since, $- - s = s \in$ TRANS, then
$$\begin{aligned}\top &= \sum_{t_1\in\mathcal{T}}\sum_{t_2\in\mathcal{T}}[(t_2 < t_1)\cdot\mathfrak{h}(s,t_1)\cdot -\mathfrak{h}(s,t_2)]\\ &= \sum_{t_1\in\mathcal{T}}\sum_{t_2\in\mathcal{T}}[(t_2 < t_1)\cdot\mathfrak{h}(- - s,t_1)\cdot \mathfrak{h}(-s,t_2)]\\ &= \sum_{t_1\in\mathcal{T}}\sum_{t_2\in\mathcal{T}}[(t_2 < t_1)\cdot -\mathfrak{h}(-s,t_1)\cdot \mathfrak{h}(-s,t_2)]\\ &= \sum_{t_2\in\mathcal{T}}\sum_{t_1\in\mathcal{T}}[(t_2 < t_1)\cdot\mathfrak{h}(-s,t_2)\cdot -\mathfrak{h}(-s,t_1)]\end{aligned}$$
Similarly, we can show that
$$\sum_{t_2\in\mathcal{T}}\sum_{t_1\in\mathcal{T}}[(t_2 < t_1)\cdot\mathfrak{h}(-s,t_2)\cdot -\mathfrak{h}(-s,t_1)] = \top$$
Hence, $-s \in$ TRANS. □

Now, the class TEMP does not have the nice closure properties that other classes enjoy; it is closed neither under $\cdot$ nor under $+$.

Lemma 1 *If $s \in$ TRANS and $s_e \in$ ETER $- \{\top\}$, then $s + s_e \in$ TEMP.*

Proof. We prove the result by showing that $s + s_e \notin$ ETER $\cup$ PERM $\cup$ CO-PERM. First, we show that $s + s_e \notin$ ETER. To that end, assume that $s + s_e \in$ ETER. Given that $s \in$ TRANS and that $s_e \in$ ETER $- \{\top\}$, we have
$$\begin{aligned}\top &= \sum_{t_1\in\mathcal{T}}\sum_{t_2\in\mathcal{T}}[(t_1 < t_2)\cdot\mathfrak{h}(s,t_1)\cdot -\mathfrak{h}(s,t_2)]\\ &\leq \sum_{t_1\in\mathcal{T}}\sum_{t_2\in\mathcal{T}}[(t_1 < t_2)\cdot\mathfrak{h}(s+s_e,t_1)\cdot -\mathfrak{h}(s,t_2)]\\ &\leq \sum_{t_1\in\mathcal{T}}\sum_{t_2\in\mathcal{T}}[(t_1 < t_2)\cdot\mathfrak{h}(s+s_e,t_2)\cdot -\mathfrak{h}(s,t_2)]\\ &= \sum_{t_1\in\mathcal{T}}\sum_{t_2\in\mathcal{T}}[(t_1 < t_2)\cdot\mathfrak{h}(s_e,t_2)\cdot -\mathfrak{h}(s,t_2)]\\ &\leq \sum_{t_1\in\mathcal{T}}\sum_{t_2\in\mathcal{T}}[(t_1 < t_2)\cdot\prod_{t\in\mathcal{T}}\mathfrak{h}(s_e,t)\cdot -\mathfrak{h}(s,t_2)]\\ &= \left[\prod_{t\in\mathcal{T}}\mathfrak{h}(s_e,t)\right]\cdot\left[\sum_{t_1\in\mathcal{T}}\sum_{t_2\in\mathcal{T}}[(t_1 < t_2)\cdot -\mathfrak{h}(s,t_2)]\right]\\ &\leq \prod_{t\in\mathcal{T}}\mathfrak{h}(s_e,t)\end{aligned}$$
But, then, $\prod_{t\in\mathcal{T}}\mathfrak{h}(s_e,t) = \top$, and, by the properties of $\mathfrak{h}$, it follows that $s_e = \top$. This is contradictory since $s_e \in$ ETER $- \{\top\}$. Hence, $s + s_e \notin$ ETER. Similarly, we can prove that $s + s_e \notin$ PERM. Likewise, $s + s_e \notin$ CO-PERM. (Although in this case we use the second clause in Definition 9.) □

Theorem 9 *There are two states $s_1, s_2 \in$ TEMP, where $s_1 \cdot s_2 \neq \bot$ and $s_1 \cdot s_2 \notin$ TEMP.*

Proof. Let $s \in$ TRANS and $s_e \in$ ETER $- \{\top, \bot\}$. By Lemma 1, $s + s_e \in$ TEMP. Given Theorem 8, $-s \in$ TRANS and by Lemma 1 again, $-s + s_e \in$ TEMP. Now take $s_1 = s + s_e$ and $s_2 = -s + s_e$. Hence, $s_1 \cdot s_2 = (s + s_e)\cdot$

$(-s + s_e) = s_e$. Since $s_e \in$ ETER $- \{\top, \bot\}$, it follows that $s_1 \cdot s_2 \neq \bot$ and $s_1 \cdot s_2 \notin$ TEMP. □

Similarly, we can prove the following.

Theorem 10 *There are two states $s_1, s_2 \in$ TEMP, where $s_1 + s_2 \neq \top$ and $s_1 + s_2 \notin$ TEMP.*

Similar to TEMP, TRANS is closed under neither $\cdot$ nor $+$. For example, the state s_1 of my running my first mile on a particular day and the state s_2 of John's running his first mile on the same day may be totally unrelated that neither $s_1 \cdot s_2$ nor $s_1 + s_2$ is guaranteed to satisfy Definition 9. The following result holds, but the proof is omitted for limitations of space

Theorem 11 *For every $\varnothing \neq \hat{S} \subseteq$ TRANS, if $\prod_{s \in \hat{S}} s \neq \bot$ ($\sum_{s \in \hat{S}} s \neq \top$), then $\prod_{s \in \hat{S}} s$ ($\sum_{s \in \hat{S}} s$) $\in$ TEMP.*

Truth

As should be clear, the semantics of $Log_A\mathbf{S}$ has no place for a notion of truth. While we can happily accommodate the standard semantic relations of consequence and equivalence and the property of logical validity, our semantic apparatus has nothing to say about truth. One question, which we will first have to answer, regards the syntactic elements of $Log_A\mathbf{S}$ of which we may claim truth or falsity. After all, we have no sentences and, unlike $Log_A\mathbf{B}$ (Ismail 2012), we have no proposition-denoting terms.

We can, nevertheless, replace the notion of truth in a world by that of holding in a world, where the latter is readily attributable to states. Whether there are differences between the two notions is the subject of extensive study in metaphysics. However, from a (commonsense) logical perspective, I do not believe that such difference pose any threats. Thus, we only need to revise the notion of a "world structure" in (Ismail 2012).

Definition 10 *For every $Log_A\mathbf{S}$ structure $\mathfrak{S}$, a bivalent world structure $\mathfrak{W}_\mathbf{2}(\mathfrak{S})$ is a countably-complete ultrafilter of $\langle$ATEMP$, +, \cdot, -, \bot, \top\rangle$.*

Intuitively, the world structure $\mathfrak{W}_\mathbf{2}(\mathfrak{S})$ comprises the set of holding atemporal states, constrained in a such a way to yield sets which agree with our intuitive interpretation of the logical connectives. (See (Burris and Sankappanavar 1982), for the exact definition of ultrafilters.) In what follows, a bivalent model of a $Log_A\mathbf{S}$ language L_Ω is a pair $\mathcal{M}_\mathbf{2} = \langle \mathcal{V}, \mathfrak{W}_\mathbf{2}(\mathfrak{S})\rangle$, where $\mathcal{V} = \langle \mathfrak{S}, \mathcal{V}_\Omega, \mathcal{V}_\Xi\rangle$ is an L_Ω valuation and $\mathfrak{W}_\mathbf{2}(\mathfrak{S})$ is a bivalent world structure for $\mathfrak{S}$. σ_S terms denoting atemporal states will, for convenience, be referred to as "sentences."

Definition 11 *A sentence ϕ is* true *in a bivalent model $\mathcal{M}_\mathbf{2} = \langle \mathcal{V}, \mathfrak{W}_\mathbf{2}(\mathfrak{S})\rangle$, denoted $Tr_{\mathcal{M}_\mathbf{2}}(\phi)$, if $[\![\phi]\!]^\mathcal{V} \in \mathfrak{W}_\mathbf{2}(\mathfrak{S})$. Otherwise, ϕ is* false *in $\mathcal{M}_\mathbf{2}$, denoted $Fl_{\mathcal{M}_\mathbf{2}}(\phi)$.*

The classical truth conditions for compound sentences follow from the above definition. (See (Ismail 2012) for the proof.)

Proposition 1 *Let L_Ω be a $Log_A\mathbf{S}$ language with a bivalent model $\mathcal{M}_\mathbf{2} = \langle \mathcal{V}, \mathfrak{W}_\mathbf{2}(\mathfrak{S})\rangle$ and let ϕ and ψ be sentences and $x \in \tau$.*

- *$Tr_{\mathcal{M}_\mathbf{2}}(\neg\phi)$ if and only if $Fl_{\mathcal{M}_\mathbf{2}}(\phi)$.*
- *$Tr_{\mathcal{M}_\mathbf{2}}(\phi \wedge \psi)$ if and only if $Tr_{\mathcal{M}_\mathbf{2}}(\phi)$ and $Tr_{\mathcal{M}_\mathbf{2}}(\psi)$.*
- *$Tr_{\mathcal{M}_\mathbf{2}}(\phi \vee \psi)$ if and only if $Tr_{\mathcal{M}_\mathbf{2}}(\phi)$ or $Tr_{\mathcal{M}_\mathbf{2}}(\psi)$.*
- *$Tr_{\mathcal{M}_\mathbf{2}}(\forall x(\phi))$ if and only if $Tr_{\mathcal{M}^b_\mathbf{2}}(\phi)$, for all $b \in \mathcal{D}_\varsigma$ (x is of type ς), where $\mathcal{M}^b_\mathbf{2}(\phi)$ is identical to $\mathcal{M}_\mathbf{2}(\phi)$ with $\mathcal{V}$ replaced by $\mathcal{V}[b/x]$.*

Conclusion

The ontology of states I have presented here is rather simple; states are taken at face value, and their proposition-like nature is manifested in their organization in a Boolean algebra. I have consciously avoided all philosophical questions related to the exact nature of states, taking them to be mere particulars, and not even committing to whether they are abstract or concrete. This is just as well; for nothing much hangs on the exact metaphysics of states if our motivation is commonsense temporal reasoning.

This being said, I have not discussed much temporal reasoning in $Log_A\mathbf{S}$. Neither have I demonstrated the expressivity of the language in representing temporal discourse. These tasks anticipate completing $Log_A\mathbf{S}$ with an account of events and a proof theory. My primary conclusion here is twofold.

First, I hope I have convinced the reader of the utility of the algebraic approach to temporal logic. Unlike most temporal logics in artificial intelligence, $Log_A\mathbf{S}$ provides a unified treatment of states and propositions. This is done by doing without propositions at all. "Sentences" in $Log_A\mathbf{S}$ are terms, albeit ones that denote atemporal states. The syntactic and ontological redundancy which one finds in first-order temporal logics endorsing states are avoided by the algebraic approach. In first-order temporal logics, reference to composite states (conjunctions thereof, for example) either is forbidden (as, for example, in the situation calculus (McCarthy and Hayes 1969)) or results in duplicating the logical connectives for statements and state-denoting terms (as, for example, in (Allen 1984).) In $Log_A\mathbf{S}$, reference to composite states is straightforward, with a single set of state-based logical connectives.

Second, I have classified states according to their temporal stability. The four intuitive classes ETER, PERM, CO-PERM, and TEMP which partition the set of states are motivated by their hosting aspectully-significant state types: states of states holding in ETER (ATEMP, in particular), past perfect states in PERM, future perfect states in CO-PERM, and progressive states in TEMP (TRANS, in particular). I have provided (original, as far as I can tell) proofs of the mostly intuitive closure and separation properties of the identified classes of states. ETER and ATEMP are themselves Boolean algebras, whereas PERM and CO-PERM are closed under finite, non-trivial sums and products, but not under complementation. On the other hand, TEMP and TRANS are closed under complementation, but not under sums and products. It is possible that such proofs may have been simpler if carried out in the object language $Log_A\mathbf{S}$, rather than at the level of semantics. This, however, requires a sound (and, preferably, complete) proof theory of $Log_A\mathbf{S}$ which, at this stage of the work, is not mature enough.

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